SEPTEMBER

NCTM® NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Five fossil hunters counted their individual finds at the end of a day's pursuit. The best mathematician among them noted that the mean number of fossils found was 13, the mode was 9, and the range was 14. What are all possibilities for the greatest number of fossils found by one of the five hunters?

1

Examine this triangular array of odd integers:

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41
⋮

What is the value of the median of the 18th row?

2

Without using your calculator, order these numbers from largest to smallest:

$$\sqrt{7},\ \sqrt{3}+1,\ \text{and}\ \frac{2\sqrt{10}}{3}$$

3

Square $MATH$ has vertices M (3, 2), A (3, 5), T (6, 5), and H (6, 2). Find an equation of the line through the origin that divides the square into two regions of equal area.

4

A sound barrier along a highway consists of 148 panels. Every 5th panel beginning with the 4th panel is sand colored. Every 3rd panel beginning with the 5th panel has an oleander shrub planted in front of it. How many of the 148 panels are sand colored but do not have an oleander bush in front?

5

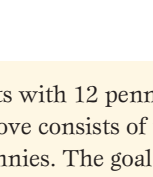
A light-year is to an astronomical unit (AU) as a mile is to an inch. This rule of thumb can help us approximate the vast distances that astronomers refer to. (An AU is slightly less than the average distance from the Earth to the sun.) If a light-year is 63,240 AU, find the percent error in the estimate of a light-year given by the mile : inch analogy.

6

Ramona gave Ramon her phone number but inadvertently dropped a digit. She wrote: 123-45-6789. Assume that the missing digit occurs between the dashes. The first digit after the first dash cannot be 0 or 1; any digit can appear after that first digit. If Ramon uses trial and error, what is the maximum number of trials he must make to ensure that he can reach Ramona?

7

Kiri folded a paper square twice. Then she made a single straight cut through all thicknesses. When she unfolded the square, she saw that she had cut out a square. How might Kiri have folded and cut the paper?



8

Find the sum of the digits of the expanded form of the integer $192^3 \cdot 5^{19}$.

9

For the following triangular array of odd integers, find a function, $f(n)$, that returns the rightmost integer in the nth row.

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41
⋮

10

If 121! were written as the product of primes, how many times would 11 appear as a factor?

11

Find all two-digit numbers such that reversing the digits results in another two-digit number that is 75% larger than the original number.

12

The children's book *Madeline*, by Ludwig Bemelmans, begins, "In an old house in Paris that was covered with vines, lived twelve little girls in two straight lines." The book's illustrations show the girls walking through Paris in an arrangement of six pairs. In how many different ways can the twelve girls be paired?

13

A positive quantity is increased by 28%. Later, the new quantity is increased by 62%. This total increase is equivalent to two equal sequential increases of p%. Find p.

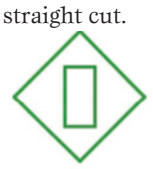
14

Find the area of the region bounded by the lines

$$y = x/2,\ y = 2x,\ \text{and}\ y = 2.$$

15

Problem 8 offered the challenge of cutting out a square from the center of a folded paper square with a single straight cut. Modify the folding procedure so that you can cut out a rectangle that is not a square with a single straight cut.



16

The sum of the lengths of the diagonals of a rhombus is 10, and the positive difference of their lengths is 4. Find the area of this rhombus.

17

Examine this triangular array of odd integers:

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41
⋮

What is the sum of the integers in the 18th row?

18

Four couples have purchased tickets to the school play. Their seats are next to one another in a single row. If each couple sits side by side, how many seating arrangements are possible?

19

A drawer contains n socks, of which 16 are striped. Two socks are pulled from the drawer in the dark. What is the largest possible value for n if the probability that both socks are striped is greater than 1/2?

20

How many ordered triples (x, y, z) are solutions to the equation $x + y + z = 8$ if x, y, and z are positive integers?

21

Triangle T has vertices A (–9, 10), B (1, 0), and C (5, 4). Points A, B, and P (also in the xy-plane) form a triangle that is congruent to triangle T. Find all possible coordinates for P.

22

P pennies are lying on a table. Of them, 1/5 show heads. If Penelope turns over 3 of the pennies showing tails, 1/4 of the pennies will show heads. How many pennies are lying on the table?

23

If we are given the arithmetic sequence –14, –11, –8, –5, …, how many terms of the sequence must be taken to obtain a sum of 30?

24

K (–1, 7), I (5, 3), and T (–1, –1) are three vertices of a concave kite. Find the coordinates of the fourth vertex if the area of the kite is 16 square units.

25

Given this triangular array of odd integers

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41
⋮

and the results from problems 10 and 18, derive the formula for the sum of the first n cubes.

26

The Complex Cookie Store sells cookies for $\$(1 - i)$ each. How much will $3 + 3i$ cookies cost?

27

A game starts with 12 pennies tails-up on a table. A move consists of turning over exactly 5 pennies. The goal is to have all 12 pennies showing heads. Is this goal achievable? If so, find the fewest number of moves required.

28

Find the value of b such that the pair of graphs $y = |x|$ and $y = -|x| + b$ enclose an area of 32 square units.

29

The 7th term of a geometric sequence is 20, and the 13th term is 11. What is the 10th term?

30